Meantone Tunings

In the course of the 19th century, Equal-Tempered Ttuning[1], **ETT**, became increasingly important compared to other tuning systems. Today almost all keyboard instruments are in 12EDO (Equal Divisions of the Octave). It has almost become a matter of course.

Obviously 1: Play and listen to various fifths on a piano or organ. With sustained sounds you can hear beatings. Do the same experiment at different positions and you will notice that the tempo of those beats differs depending on whether you play those sounds higher or lower on the keyboard.

Obviously 2: play the circle of fifths. Start with the lowest C on your keyboard:

C	G	D	A	E	B	F#	C#	G#	D#	A#	F	C
0	1	2	3	4	5	6	7	8	9	10	11	12

At the end you get a C that is exactly 7 octaves higher. However historically this tuning system is made up of **perfect** fifths. The fifths you just played are usually called perfect, but well considered they aren’t at all. They will only become so if you would tune them in such a way that the beatings have totally disappeared and you hear a harmony that is completely at rest.

Pythagorean tuning

In the Middle Ages, the most common tuning system for keyboard instruments was the one based on perfect fifths. This tuning is also called Pythagorean tuning named after Pythagoras, who formed the basis of this tonal system. However, at the end of the circle of perfect fifths we end up with a quasi C considerably too high! The difference lies in the fact that the perfect C is a result of stacking 7 octaves and therefore has the frequency ratio of $2^7 = 128$, while stacking 12 pure fifths gives the quasi C with a frequency ratio of $(2/3)^{12} = 129.7463379$. Converted to cents, that gives 24 cents rounded. 12 equal tempered fifths of 700 cents stacked, yield 8400 cents, while 12 pure fifths of 702 cents gives 8424 cents.

In the **ETT** we spread this small interval, the Pythagorean comma, equally over all 12 intervals in the circle of fifths. As a result, when we start the series at C after a stack of 12 fifths, we come back to a C that is exactly 7 octaves higher. We have now obtained all tones of the ETT, albeit that they are spread over 7 octaves. Octave transposing brings this 12-tone series within a one octave range. This **ETT** is therefore based on 1/12 division of the Pythagorean comma.

The historical 1/4’ meantone tuning

A well-known historical tuning is the 1/4’ meantone tuning, which is also simply called simply meantone tuning. In this temperament the starting point is the **perfect** major third. After stacking four perfect fifths on C: G D A E, we obtain the **Pythagorean** major third E, two octaves higher than the initial tone C (four perfect fifth intervals stacked result in $(3/2)^4 = 5.0625$). However, the **perfect** major third E, two octaves higher, has the ratio 5. The interval between the two major thirds is found by division 5.0625/5 = 1.0125, which converted[3] corresponds to 21.5 cents. This small interval, the discrepancy between the mean major third and the perfect major third, is called the **syntonic** comma or the comma of **Didymus** after his discoverer. The small, barely perceptible, interval between the Pythagorean comma and the syntonic comma is called **schism** and is 1.9537 cents.

The distribution of the syntonic comma

In the 1/4’ meantone temperament, the 21.5 cent syntonic comma is equally spread over the stack of fifths. The fourth fifth in the stack is, as we saw, the too high Pythagorean major third E. If we now reduce each fifth interval by 1/4 comma, we change the high Pythagorean E to a pure E. 1/4 syntonic comma corresponds to a 5,375 (21.5/4) cent interval. Each tempered fifth interval now amounts 702-5.375 = 696.625 cents, rounded to 697 cents. A stack of 11 of these fifths gives 11 times 697 is equal to 7667 cents. Finally, an interval of 8400-7667 = 733 cents remains!
So that is a far too large ‘fifth’, a screaming ‘false’ interval and therefore called the wolf's fifth.

Features of the 1/4' meantone temperament

Characteristic are the pure major thirds and pure minor sixths. However, this is at the expense of the no longer perfect – slightly small – fifths, which consequently result in a slow beating. That this temperature became popular in history is not surprising. When, at the end of the 15th century, the major third was increasingly perceived as a consonant, it was then the usual Pythagorean tuning, with impure major thirds, that disappeared in favor of the 1/4' meantone temperament with pure major thirds. The name meantone owes this temperament to the fact that the major second is exactly in the middle of the major third. The nice quiet major thirds and minor sixths have the aforementioned disadvantage, the wolf fifth. This wolf interval is traditionally not placed at the end of the cycle, but on **G#–D#**. In addition to the wolf fifth, additional wolf harmonies are created. Due to the wolf fifth and the other 'wolves' not all keys can be used. The usable ones: B flat, F, C, G, D and A; melodic: g, d, a, e, b and f#; harmonic: g, d and a.

As stated above, the 1/4' distribution is based on major thirds and minor sixths with frequency ratios 5/4 and 8/5, respectively.

1/4' meantone

Keys	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
Cents	0	76	193.2	**310.3**	**386.3**	503.4	579.5	696.6	772.6	889.7	1006.8	1082.9	(1200)
Diff 12-EDO	0	-24	-6.8	10.3	-13.7	3.4	-20.5	-3.4	-27.4	-10.3	6.8	-17.1	(0)

Profile 1/4' meantone

M3	Cents	m3	Cents	P5	Cents
C-E	386	C-D#	310	C-G	697
C#-F	427	C#-E	310	C#-G#	697
D-F#	386	D-F	310	D-A	697
D3-G	386	D#-F#	269	D#-A#	697
E-G#	386	E-G	310	E-B	697
F-A	386	F-G#	269	F-C	697
F#-A#	427	F#-A	310	F#-C#	697
G-B	386	G-B	310	G-D	697
G#-C	427	G#-B	310	G#-D#	738
A-C#	386	A-C	310	A-E	697
A#-D	386	A#-D#	269	A#-F	697
B-D#	427	B-D	310	B-F#	697

M3: major thirds; m3: minor thirds; P5: fifths
Red: wolfs

¼' meantone temperament calculated from Equal Temperament Tuning (12EDO)

Because we now start from the Equal Temperament, 12EDO, which stands for 12 Equal Divisions of the Octave, the calculation becomes somewhat simpler. Starting from the perfect major third with a frequency ratio of 5:4, we obtain the interval size in cents via the formula ln(5:4):ln2x1200=386. This perfect major third is 14 cents (400–386) lower than the Equal Temperament.
We reduce the fifths by ¼ of this difference: 700 – (14:4) = 3.5. Instead fifths of 700 cents, we now obtain tempered fifths with intervals of 696.5 cents.
Next, we build the circle of fifths up to cycle position 8 by stacks of 696.5 cents:

C	0		
G	696,5		0
D	1393	> -1200	= 193
A	2089,5	> -1200	= 889,5
E	2786	> -2400	= 386
B	3482,5	> -2400	= 1082,5
F#	4179	> -3600	= 579
C#	4875,5	> -4800	= 75,5

Wolf position #8 + 738,5

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G#    5572        > -4800 = 772
D#    6310        > -6000 = 310
A#    7007        > -6000 = 1007
F     7703,5      > -7200 = 503
(C)   (8400)
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In the 8th stacking position (G#-D#), we add the *wolf fifth by adding 738.5* cents instead of a fifth of 696.5 cents. Then, as before, extend the circle of fifths with the tempered fifths of 696.5 cents.

In order and rounded:

C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
0	76	193	**310**	**386**	503	569	697	772	890	1007	1083	1200

Other distributions of the syntonic comma

Also on the basis of the syntonic comma, a large number of variants of the 1/4' meantone temperament have emerged throughout history. On the one hand by moving the wolf fifth and also by dividing the 'falsehood' of the 'wolf' over, for example, several fifths intervals. On the other hand, by applying an increasingly smaller part of the comma as a distribution amount, for example: 1/3', 1/5' and 1/6' meantone. 1/11 division of the syntonic comma results in a theoretical almost perfect approximation of 1/12 division of the Pythagorean comma. Practically this difference is negligible.

1/5' meantone temperament

Keys	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
Cents	**0**	83.6	195.3	**307**	**390.6**	502.3	585.9	697.7	781.2	893	1004.7	1088.3	(1200)
Diff 12-EDO	0,	-16.4	-4.7	7	-9.4	2.3	-14.1	-2.3	-18.8	-7	4.7	-11.7	(0)

Profile 1/5' meantone

M3	Centa	m3	Cents	P5	Cents
C-E	391	C-D#	307	C-G	698
C#-F	419	C#-E	307	C#-G#	698
D-F#	391	D-F	307	D-A	698
D#-G	391	D#-F#	279	D#-A#	698
E-G#	391	E-G	307	E-B	698
F-A	391	F-G#	279	F-C	698
F#-A#	419	F#-A	307	F#-C#	698
G-B	391	G-B	307	G-D	698
G#-C	419	G#-B	307	G#-D#	726
A-C#	391	A-C	307	A-E	698
A#-D	391	A#-D#	279	A#-F	698
B-Eb	419	B-D	307	B-F#	698

1/6' meantone temperament

Keys	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
Cents	0	88.6	196.7	**304.9**	**393.5**	501.6	590.2	698.4	787	895.1	1003.3	1091.9	(1200)
Diff 12-EDO	0,	-11.4	-3.3	4.9	-6.5	1.6	-9.8	-1.6	-13	-4.9	3.3	-8.1	(0)

Profile 1/6' meantone

M3	Cents	m3	Cents	P5	Cents
C-E	394	C-D#	305	C-G	698
C#-F	413	C#-E	305	C#-G#	698
D-F#	394	D-F	305	D-A	698

D#-G	394	D#-F#	285	D#-A#	698
E-G#	394	E-G	305	E-B	698
F-A	394	F-G#	285	F-C	698
F#-A#	413	F#-A	305	F#-C#	698
G-B	394	G-B	305	G-D	698
G#-C	413	G#-B	305	G#-D#	718
A-C#	394	A-C	305	A-E	698
A#-D	394	A#-D#	285	A#-F	698
B-D#	413	B-D	305	B-F#	698

1/3' meantone temperament

Keys	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
Cents	0	63.5	189.6	315.6	379.1	505.2	568.7	694.8	758.3	884.4	1010.4	1073.9	(1200)
Diff 12-EDO	0,	-36.5	-10.4	15.6	-20.9	5.2	-31.3	-5.2	-41.7	-15.6	10.4	-26.1	(0)

Profile 1/3' meantone

M3	Cents	m3	Cents	P5	Cents
C-E	379	C-D#	316	C-G	698
C#-F	413	C#-E	316	C#-G#	698
D-F#	379	D-F	316	D-A	698
D#-G	379	D#-F#	253	D#-A#	698
E-G#	379	E-G	316	E-B	698
F-A	379	F-G#	253	F-C	698
F#-A#	413	F#-A	316	F#-C#	698
G-B	379	G-B	316	G-D	698
G#-C	413	G#-B	316	G#-D#	757
A-C#	379	A-C	316	A-E	698
A#-D	379	A#-D#	253	A#-F	698
B-D#	413	B-D	316	B-F#	698

Attention

All these temperaments (for keyboard instruments!) have the implicit starting point that the relevant sound generators have a harmonic overtone structure (harpsichords, organs). That means that the frequencies of the overtones are all whole multiples of the fundamental frequency. The frequencies in the sound spectrum of the tones relate as 1 (fundamental), 2, 3 etc. This in combination with the mutual coordination of sound sources (vibrating strings and air columns) in various tuning systems causes problems.

For example

Suppose an organ in ETT. Choose diapason 8 'and play the C4-G4' fifth interval. You don't even have to listen in a concentrated way to notice that this harmony results in a beating. The cause lies in the fixed fact that the overtones of the diapason stop are whole multiples of the fundamental frequency. In other words, the overtones (the internal tuning, so to speak) are in just intonation; they therefore relate in frequency as: 1, 2, 3, ... Nevertheless, in the C4-G4 keys relation, the G4 diapason pipe' is in ETT ratio to the C1 diapason pipe. This has the following consequence: the third harmonic of the C4 diapason does not exactly coincide with the second harmonic of the G4 diapason. Simply due to the fact that the ETT fifth compared to the perfect fifth is a fraction too small. Nothing can be changed about the just intonation in the overtone series of vibrating air columns and (ideal) strings. The only option there is to reconcile the **fundamental discrepancy** between pure overtones and external tuning is an adjustment of the external tuning, for example: 1/4 'meantone, Werckmeister, Valotti, Young, ETT etc.

Idiophones

With idiophones, instruments that produce tones with a non-harmonic or only partially (quasi) harmonic overtone structure, the importance of which tuning and why must be seen in a different perspective. Certainly for those instruments that can be adjusted in the tuning of the overtones (bells, metallophones, marimbas etc.). The most important condition, however, is that *the internal overtone structure must be, if possible, in accordance with the external tuning of the sound generators*. An example from the carillon world. Carillon bells are very characteristic in sounding a minor third in their overtones. If you play a C 'you will get an extra Eb sounding. The bell caster for instance has tuned the bells for a carillon in the ETT. Because of this ETT, the minor third is more precisely defined as ETT third, which is thus in accordance with the tuning of the bells relative to each other. If you play the interval C4-Eb4 then the prime partial (the name of the firtst overtone in the bell sound) of the Eb4 bell exactly matches the minor third (the name of the third overtone in the bell sound) of the C4 bell.

Bowed and wind instruments

Freely intonating musicians, players of bowed instruments and wind instruments, can intonate lower or higher as desired and they do so intuitively. The whole tuning issue is really only the problem of our keyboard instruments, which have only a limited number of physical keys and therefore a certain tuning choice has to be made. Again this stems from the fact that the tones of the sound generators, vibrating (ideal) strings and vibrating air columns, have a fixed harmonic overtone structure.

Pitch and frequency

Pitch and frequency, two concepts that show a special relationship. Pitch is a psychological quality, while frequency represents a physical quantity. We will take a closer look at the relationship between these two concepts on the basis of an example. We immediately recognize a certain melody as the same whether it is played in a high or low position. We distinguish intervals: octaves, fifths, thirds, etc. If we look at the frequency differences of two identical melodies, one in low position and the other variant in high position, it appears that the successive frequency differences in both variants are completely different. A closer look shows that the successive intervals, frequency ratios, are identical. A linear relation of pitch sensation requires therefore a geometric relation of frequency. This is how we recognize the exponential 100, 200, 400 and 800Hz frequency series, in which each frequency is successively multiplied by 2, as a series of four consecutive octaves.

[1]Internationally, this tuning system is often referred to as 12-EDO (12 Equal Divisions of the Octave). You also come across names like 12-TET and 12-ET, abbreviations of 12- (**T**one) **E**qual **T**emperament.

[2]**The cents system**
The English mathematician Alexander Ellis designed a system for determining musical intervals that focuses on the linearity of pitch perception. He divided the octave into 1200 equal micro intervals. An interval of a semitone is thus by definition equal to 100 cents. Nowadays this cent system is the standard for (scientific) musical interval notation. The already mentioned small interval, known as the Pythagorean comma, measures around 24 cents according to this system, which amounts to almost the eighth part of a whole tone. With the following formulas you can easily calculate frequency ratios to cents and vice versa.

[3]**Calculation ratios and intervals**

Starting from **R** between two frequencies (xHz/yHz) you get the interval **I** in cent: $I=\ln R/\ln 2 \times 1200$

Conversely, from the interval **I** to the decimal frequency ratio **R**: $R=e^{(I \times \ln 2/1200)}$

Hz is the unit of frequency and by definition 1Hz=1 vibration cycle per second

Consulted literature and further reading

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Dover Publications, Inc. Mineola, New York

Campanologie
dr. André Lehr
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https://www.historicaltuning.com/TuningsSummary.html

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http://www-personal.umich.edu/~bpl/larips/meantone.html

'Mean Tone Scale Generator'

https://robertinventor.com/software/tunesmithy/help/mean_tone_in_cents.htm

https://www.yacavone.net/xen-calc/?q=3^%281/19%29

Internet, many articles on Wikipedia, key words: tuning, meantone, comma, Pythagorean, wolf interval etc.