Footages vs Harmonics

In tradition the past comes back to life. For example, the footage designation, indicated by a number followed by a comma in superscript, traditionally indicates the length - in the number of feet - of an organ pipe. However, on many hardware or software synthesizers we still find a physical button or parameter where we can choose between, for example, 16 ', 8' or 4 '. Selecting one of these options results in transpositions of an octave. 16 'sounds an octave lower than 8'. 4' one octave higher than 8'.

How come?
We know from physics that the length of a vibrating air column is inversely proportional to the periodicity of the vibration. Periodicity defined as the number of vibration cycles per second. This is expressed in hertz, abbreviated Hz. By definition, 1 vibration period per second is equal to 1 Hz. If we double the length of the air column, it appears that the periodicity is halved. However, if we halve the length of the organ pipe, the periodicity appears to have increased by a factor of 2. Our perception recognizes periodicity ratios. A factor of 2 results in an octave for our perception.

The fundament of the organ
The organ includes a *polyphonic* keyboard instrument with at least one entirely independent tone generator for each key (the smallest instrument in this category is the 'Portative'). One such set of generators coupled to the keyboard is called a stop. An organ consists of multiple stops in various differentiated footages. In most cases these stops can be switched on and off as desired. In this way, several independent sources can be sounded by one key.

Footages
The so-called 8' stop produces the same pitch as the comparative key on the piano. If we now determine the length of the open pipe (open flute) at lowest the C key on the keyboard, it turns out to be approximately 240 cm. This register, this flute-like collection, is called an 8 'stop. One foot is equal to 30 cm; 8' thus corresponds to 2.40 m. A 4' stop therefore sounds an octave higher than one would expect on the basis of the test key.

Open and closed pipes
On an organ we find so-called closed pipes next to open pipes. These are flutes with the top of the tube closed. At the same length, a covered pipe appears to sound an octave lower than an open flute. Here too we find the explanation in physics. Even if the length of such a register sounding at the same pitch is only half the size of that of the open flutes, the stop is labeled as 8 'register.
The sound of the open pipes consists of a consecutive harmonic series: both even and odd harmonics. The narrower the diameter, the more overtones. The same applies to the closed pipes, with the difference that they only produce odd harmonics.

Various footages
The best known stop footages on pipe organs are 16 ', $5^{1/3'}$, 8', 4', $2^{2/3}$', 2', $1^{3/5}$', $1^{1/3'}$ and 1'. In this order we also find them on the well-known Hammond organ. Why exactly these footages: expressed in whole as well as broken numbers? That has everything to do with the vibration produced in such a pipe; this concerns a so-called periodic complex vibration. Each complex vibration consists of a number of sub-frequencies, sinusoidal vibrations ('the sound atoms'). For a complex periodic vibration, the constituent partial frequencies relate to the following sequence: 1, 2, 3, 4, 5, 6, 7, 8 ... This spectrum of frequencies is also called the fundamental tone with its overtones. The lowest frequency the '1' is the fundamental, the other, the multiples, are the overtones. This series is known as the harmonic series, or also called nature tone series.

The relationship between footages and harmonics
We compare the foot size range 8 ', 4', $2^{2/3'}$, 2', $1^{3/5'}$, $1^{1/3'}$, $1^{1/7'}$, 1' with the following harmonic sequence: 1, 2, 3, 4, 5, 6, 7, 8, it appears numerically to be the mirror image of the foot numbers.

In formula form for an 8 'register: 8 divided by the foot measure number gives the harmonic number. For example, the $2^{2/3'}$ appears to be the third harmonic of the 8' register: $8:2^{2/3} = 3$. For example, we find the foot size for the third harmonic of a 16 'register by dividing 16 by 3, $16:3 = 5^{1/3}$. The above footages range thus appears to represent nothing else than a harmonic series. Instead of directly indicating the periodicity ratios, it refers to its cause, the length ratios of the vibrating air columns.

The Equal Tempered Tuning (ETT)
For polyphonic keyboard instruments, the ETT, also called equal temperament, is defined with all successive semitones being in mutual frequency relation by the 12th root of 2 (1.059463094). This tuning has become standard in western music practice. Suppose we have selected two stops on an organ, the 8' and the $1^{3/5'}$. For example, if we play the interval C4-E6, then the 5th harmonic of the 8' almost coincides with the fundamental of the E6 that sounds through the $1^{3/5'}$ footage at the key E6.

Discrepancy of internal tuning and external tuning
Because both do not have exactly the same pitch, there is a beating in the resulting sound. The cause of this is the discrepancy between the harmonic tuning in the overtone series of the organ pipes, the internal tuning, and the external tuning, the frequency relation of the pipes according to their difference frequency. Such a similar kind of beating also occurs with the interval C4-G5. The 3rd harmonic of the organ pipe on the C4 key is not exactly equal to the 1st harmonic, the fundamental, of the pipe on the G5 key. Only the octave footages, 8', 4', 2', … correspond to the harmonics of the overtone sequence and will not result in beating when sounding together.

Beatings
Are those beatings unpleasant or pleasant you may wonder. The answer is twofold. Both can be the case. It has everything to do with the speed of such beatings. Quite slow beatings up to about twice a second is usually experienced as pleasant. We interpret them as a kind of chorus effect. The effect that we observe when, for example, a number of violins playing unison. Faster beatings, up to about seven times per second, occur as a sort of tremolo effect. Faster beatings are even more and more annoying. However, if there are not too loud overtones and the speed of the beatings is limited, it's not a problem.

An organ with harmonic tone generators
In the case of an organ composed of sources with a harmonic overtone structure, therefore, acceptable beatings will always have to be found in the external tuning, since the harmonic overtone structure is a given. In the course of time a large number of different moods (temperatures) have been developed, which have both advantages and disadvantages.

Hammond: an organ with tone wheels in ETT
Assuming that the tone wheels generate pure (sinusoidal) frequencies that function both as fundamental tones and overtones, the internal tuning is identical to the external tuning. For example, separate stops such as fifth, third and seventh function as quasi-harmonics. For example, the C4 key can also be linked to the tone wheel one octave higher. This link then becomes the 4' stop. Connected to the tone-wheel a twelfth higher we obtain a $2^{2/3'}$ stop, etc.

In addition to the standard set of footages:
16, '$5^{1/3'}$, 8', 4', $2^{2/3'}$, 2', $1^{3/5'}$, $1^{1/3'}$, 1 '
We find two additional drawbars on the Hammond H100, both of which represent a combination of two quasi-harmonics: the 7th ($1^{1/7'}$) and 9th (8/9') combination, and an assembly of the 10th (4/5 ') and 12th (2/3'). Ascending in the footage range we see that the deviation is increasing. The fifths are 1.9 cents up, the 8/9 'is 3.9 cents high, the third size $1^{3/5'}$ 13.7 cents too high, the seventh foot size $1^{1/7'}$ differs even more, this one is 31.2 cents too low.

For the sake of convenience, I assumed that the Hammond tone-wheel organ would be made up of tone wheels that are in exactly equal tempered tuning relation. And that these tone wheels would also produce exactly sinusoidal signals. These two assumptions are incorrect. In both cases it concerns an approach. However, the deviations are small enough to be irrelevant for the scope of this article. Just so that I take into account that the high footages fold back. Again, this is not essential for the scope of this article.

Standard Hammond footages

(quasi)harmonics in ETT with 8' reference)

Footages	16'	$5^{1/3}$'	8'	4'	$2^{2/3}$'	2'	$1^{3/5}$'	$1^{1/3}$'	1'
harmonics	0.5	1.5	**1**	2	3	4	5	6	8
Interval in cents	-1200	-500	**0**	1200	1900	2400	2800	3100	3600
Offset in cents with harmonic	0	1.9	**0**	0	1.9	0	13.7	1.9	0

Extra Hammond H100 footage combinations

Footages	$1^{1/7}$'	8/9'	4/5'	2/3'
quasi-harmonics	7	9	10	12
Interval in cents	3400	3800	4000	4700
Offset in cents with harmonics	-31.2	3.9	13.7	1.9

Basic footage : stop footage = harmonic number Basic footage : harmonic number = stop footage

Hammond Console (A, B, C) Factory Presets Upper Manual

Preset Key	Drawbar settings	Name	Loudness
C	-	-	No sound
C#	005320000	Stopped Flute	pp
D	004432000	Dulciana	ppp
D#	008740000	French Horn	mf
E	004544222	Salicional	pp
F	005403000	Flutes 8' & 4'	p
F#	004675300	Oboe	mf
G	005644320	Swell Diapason	mf
G#	006876540	Trumpet	f
A	327645222	Full Swell	ff
A#	Upper Left Drawbars		
B	Upper Right Drawbars With Percussion		

Hammond Console (A, B, C) Factory Presets Lower Manual

Preset Key	Drawbar settings	Name	Loudness
C	-	-	No sound
C#	004545440	Cello	mp
D	004423220	Flute & String	mp
D#	007373430	Clarinet	mf
E	004544220	Diapason, Gamba & Flute	mf
F	006644322	Great without reeds	f
F#	005642200	Open Diapason	f
G	006845433	Full Great	fff
G#	008030000	Tibia Clausa	f
A	427866244	Full Great with 16'	fff
A#	Upper Left Drawbars		
B	Upper Right Drawbars		

Popular Hammond drawbar registrations

Name	Drawbar settings	Percussion		
Blues solo	888000000	3rd	fast	soft
Solo	800000300	2nd	slow	soft
Bossanova solo	800800000	2nd	fast	soft
Blues strings	800000888	off		
Blues horns	880080888	off		
Strings 8’	004688888	off		
Open flute	008300000	off		
Stopped flute	008030000	off		
Clarinet 8’	005070300	off		
Trumpet 8’	006876540	off		
Stopped flute	005320000	off		
Auger	888110000	2nd	fast	soft
A whiter shade of pale	688600000	2nd	fast	soft

List of some organ mutation stops on the pipe organ (after Wikipedia)

Footage	Sounding note at C4	Name
$6^{2/5'}$	E4	Gross Tierce
$5^{1/3'}$	G4	Quint
$3^{1/5'}$	E5	Gross Tierce, Tierce
$2^{2/3'}$	G5	Nasard, Twelfth
$1^{3/5'}$	E6	Tierce
$1^{1/3'}$	G6	Larigot
$1^{1/7'}$	Bb6	Septième
8/9’	D7	None
8/13’	A7	Tredezime
8/14’	Bb7	Septième
4/9’	D8	Mollterz (Minor third)
1/6’	G9	Quadragesima

Yamaha DX synthesizers

The DX series synthesizers from Yamaha offer not only extensive FM synthesis capabilities, but also an additive synthesis mode: for the 6-operator instruments algorithm 32 and for the 4-operator instruments algorithm 8. These operators function as a mix of sine-wave generators and can be used as footages for organ registration. Below a comparison chart of drawbar volume settings related to the corresponding operator output levels.

Hammond drawbar settings versus Yamaha DX synthesizer operator output levels

Drawbar settings	DX1/7/5/TX7/DX7II/TX802 additive algorithm 32	DX11/TX81Z/V50/DX100 additive algorithm 8
1	71	62
2	75	66
3	79	70
4	83	74
5	87	78
6	91	82
7	95	87
8	99	92

References

Hammond H100, Klaus Wunderlich
https://www.youtube.com/watch?v=8vqc1GBdJcs
https://www.youtube.com/watch?v=MonIsxNMPv8

Organ/Keyboard Drawbar Basics
https://www.youtube.com/watch?v=FeRTwh9oXxo

List of organ stops
https://en.wikipedia.org/wiki/List_of_pipe_organ_stops

Basic Theatre Organ Registrations Tutorial by Tom Horton
https://www.youtube.com/watch?v=jTLkGZu2Bh8

Guide to the cinema organ by Tom Horton
https://www.youtube.com/watch?v=LEKIDS3B-O8