The FM Synthesis of a Carillon Bell

Frequency Modulation Synthesis (FM)
The American composer John Chowning discovered that the frequency modulation technique could also be used as a method for sound generation. Instead of modulating a high frequency with a low-frequency audio signal, such as in radio technology, Chowning used two audio signals: one as carrier (c), and one as modulator (m). The carrier oscillator is gradually modulated equally upwards and downwards by the modulator oscillator.

If this modulation happens slowly, we hear it as a siren or vibrato, depending on the speed and depth of that modulation. However, modulations faster than 20 times per second can no longer be perceived as such. We now experience this frequency modulation as a newly formed sound. Remarkable in the application of frequency modulation for sound generation is that often the modulation frequency is higher than the carrier or carrier frequency. Chowning applied an old known principle for a completely new application, sound synthesis.

"1 + 1 = many"
In this FM model, both oscillators produce a so-called pure tone, a sine wave, a single frequency. According to Fourier's mathematical theory, however, it can be shown that we have now obtained a resulting complex vibration consisting of a multiplicity of frequencies: a fundamental frequency with overtones.
Herein lies the power and elegance of the FM model. Starting from only two frequencies, sine waves, a new complex oscillation is generated which, can consist of a sum of many newly formed sine waves, created by the frequency modulation principle.

Timbre: the dynamics of fundamental and overtones
Each periodic sound can be decomposed into a plurality of sinusoidal vibrations, with associated frequency, amplitude and phase (which is of much less importance). The vibration with the lowest frequency is called root, first harmonic, or fundamental, the higher frequencies are called overtones, partials or harmonics. Harmonics are called those overtones that concern a whole multiple of the lowest frequency, the fundamental. Frequencies that are not integer multiples of the fundamental are referred to as partials or inharmonic frequencies.

The pitch interval between carrier and modulator determines: *which overtones*

The amount of frequency modulation determines: *how many overtones*
This is controlled by the amount of the modulator's vibration response. In other words, by the amplitude, the output volume of the modulator. Overall, the strength of the formed partials decreases upwards in the series.

Dynamics: oscillator plus envelope generator
Each oscillator, both carrier and modulator, is equipped with a so-called envelope generator, which regulates the dynamics. The output level, the amplitude, can thus be controlled independently for each oscillator.

Velocity and key scaling
The output level can also be influenced by the velocity of a key on a keyboard. The envelope generator of the carrier thus determines the actual audible signal, the ultimate loudness. The modulator envelope controls the course of the timbre. The time segments of these two envelopes can be scaled on the basis of the pitch, which key on the keyboard. The importance of these envelopes can hardly

be overestimated. Together with the spectrum, which overtones, they are very decisive for the ultimate sound identity.

The upper sideband frequencies

We obtain the upper sidebands as follows. We take the carrier number and add the modulator number to it. Then we add the modulator number to that result. Then again, again, and so on. For a c:m ratio of 2:7 as in the 'Tubular Bell' preset in the DX7, this produces frequencies with the following ratios:

(2 + 7 =) 9, (9 + 7) = 16, (16 + 7) = 23, (23 + 7) = 30

The lower sideband frequencies

In a similar way we find the lower sidebands.
First we subtract the modulator number from the carrier number. As a result for the difference frequency we now get -5. Then we neglect the minus sign. Then we repeat the same procedure as in the calculation of the upper sidebands. We then obtain the following frequencies:

(| 2-7 |) = 5, (5 + 7) = 12, (12 + 7) = 19, (19 + 7) = 26

It is also remarkable that the modulator frequency itself is lacking in the formed sound, as well as all multiples of it.

With all frequency ratios in sequence, we obtain the following overtone spectrum:

2	5	9	12	16	19	26	30

The lowest frequency '2' is the reference for our perception, or in other words: we experience it as the fundamental tone. This is interpreted by our perception as a '1'. Therefore, everything divided by 2:

1	2.5	4.5	6	8	9.5	13	15

This corresponds to the following relative tone sequence on C4:

C4	**E5**	**D6**	**G6**	**C7**	**D#7**	**A7**	**B7**

A spectrum with clearly a mixture of on the one hand harmonic sub-tones, the whole numbers, and on the other hand the inharmonic sub-frequencies, the tones with decimal numbers. From psycho-acoustics we know that a consonance of harmonic overtones tends towards 'fusion' to an indivisible sound with unambiguous pitch.

Fusion increases as the ratio numbers of the constituent frequencies are small and they form a closed series. In this example, the harmonics in the spectrum, 1, 6, 8, 13 and 15 will not really merge into an integer sound because of the lack of the lower harmonics on the one hand and, on the other hand, by the large numbers for the higher overtones. The fact that they do not form a closed series also contributes to this.

In short, the result produces an unmistakable bell-like sound with characteristics of fusion: a clear unambiguous pitch impression. But also with characteristics of 'splitting': in addition to that unambiguous pitch experience, we also hear several more or less independent tones with one that clearly stands out. That is in this case the first overtone (2.5), which is a major tenth above the strike tone.

Sound characteristics of a carillon bell

The strike tone

An important perception aspect of a carillon bell is the so-called strike tone. That is the metallic sound at the moment of the attack that determines the pitch impression. Remarkably, this pitch sensation does not necessarily have to match one of the partial tones or the fundamental tone.

It is a psychoacoustic phenomenon. If we simultaneously hear pure tones, sine tones, with simple, or approximate, harmonic frequency relationships, our perception will experience a pitch sensation on the greatest common denominator of these relationships. (This phenomenon is called residual pitch, or also called 'missing fundamental'. It was discovered in 1939 by the Dutch biophysicist Jan Schouten.) At the moment of attack the octave, the fifteenth and the triple octave sound loud, which form a frequency ratio of 2:3:4. The pitch we now experience is the (virtual) 1. This is the pitch after which we name the bell. However, the fundamental is one octave lower.

The overtone spectrum

In the table 'Sound analysis of a carillon bell' we see the constituent partials from 1 to 40. A number of bell partials have been given fixed names in the carillon world, these are also included in the table. In addition, we see the initial volume at the attack and the right column gives an impression of the relative decay times.

The strongest overtones are bold, which of course suggests that these are also the most important for the sound identity. If we hear such a bell at a great distance, it is still unmistakably a bell, while only the very strong partial frequencies are heard, the others are not perceived, they are below the hearing threshold because of the large listening distance.

The loudest frequencies in the carillon bell spectrum

(After André Lehr from: 'Leerboek der Campanologie')
Respectively the frequency ratio (idealized rounded), the partial name, the pitch (with omission of the deviation in cents), the initial loudness and the decay time.

Frequency ratio	Partial Name	Pitch	Volume	Decay time%
1	fundamental	C4	mf	100%
2	prime	C5	f	55
2.38	minor third	D#5	f	75
4	octave	C6	fff	30
6	fifteenth	G6	ff	20
8	double octave	C7	f	15
11	fourteenth	F7	mf	10
13		A7	mp	7.5
16	triple octave	C8	mp	5

Piet van Egmond realized bell imitations on the organ in the sixties of the last century. In the table below we see that he used only four organ stops for this purpose. Two very overtone-poor stops, wide open flutes for fundamental and minor tenth, and two overtone-rich stops, diapasons for the octave and the fifteenth. Together with the reverberation in the church and the way of playing: keynote and minor tenth sustained and provided with tremulant and rhythmically played as 'bim-bam', that delivered, certainly in that time, spectacular results.

The combination of stops Piet van Egmond used as bell imitation on the organ (according to my own aural analysis)

fundamental	C4	wide open flute
minor third	D#5	wide open flute
octave	C6	diapason
fifteenth	G6	diapason

Van Egmond's abstraction emphasizes once again the very characteristic aspect of the minor third (above the strike tone) in the sound.

A simple FM bell synthesis

A simple solution for an FM bell simulation consists of four oscillators: two times a pair: carrier and modulator. Both carrier outputs are simply mixed together in the desired ratio. One pair with c: m ratio 1:3 and another pair with c: m ratio 2.38:8.38. (If we simplify this last ratio, there is actually 1:3.52, which is very similar to the c:m ratio 1:3.5 (simplification of 2:7) as we already encountered in the DX7 preset 'Tubular Bell'.) Both spectra therefore show a large degree of similarity, however the spectrum formed by the c: m ratio 2.38:8.38 is in its entirety a minor tenth (1:2.38) higher than the spectrum formed by c: m ratio 1:3.

The sideband frequencies generated by a FM pair with a c: m ratio of 1:3

Partial ratio	Offset: semitones.cents	Note name.cents	Name bell partial
1	0	C4	fundamental
2	12	C5	prime
4	24	C6	octave
5	27.86	E6 -14 l	major tenth
7	33.69	A#6-31	
8	36	C7	double octave
10	39.86	E7 -14	
11	41.51	F#7-49	
13	44.40	A7 +40	
14	45.69	A#7-31	
16	48	C8	triple octave
17	49.05	C#8 +5	
19	51	D#8	
20	51.86	E8 -14	
22	53.51	F#8-49	

The pair with the ratio 2.38: 8.38 results in the following two lowest sub-frequencies: the carrier itself, 2.38 and the first difference frequency, 6. Partial 2.38 is now the minor tenth in the spectrum. In clockwise terms the minor third mentioned because it is a minor third above the tone, the virtual '1'. (Ideally, the pitch of the strike tone will coincide with that of the prime.) An overtone with a ratio of 6 is the fifteenth in the bell sound. In the table below you will also find the other formed partial frequencies.

The sideband frequencies formed by an FM pair with a c: m ratio 2.38:8.38

Partial ratio	Offset: semitones.cents	Note name.cents	Name bell partial
2.38	15	D#5	minor third
6	31.02	G6 +2	fifteenth
10.76	41.13	F7 +13	double fourteenth
14.38	46.15	A#7 + 15	
19.14	51.10	D#8 + 10	
22.76	54.10	F#8 + 10	

27.52	57.39	A8 +39
31.14	59.53	C9 -4
35.90	62	D10
39.52	63.65	E10 -35

The combination of these two above-mentioned spectra yields a total sound that is very similar to the sound of a real carillon clock. The fifth is missing, but that is not so striking because of the modest loudness mp. And instead of a B6 we see and A#6 in the spectrum.

The two spectra combined: the 4-oscillator FM bell synthesis

Partial ratio	Offset: semitones.cents	Note name.cents	Name bell partial
1	0	C4	fundamental
2	12	C5	prime
2.38	15	D#5	minor third
4	24	C6	octave
5	27.86	E6-14	major tenth
6	31.02	G6 +2	fifteenth
7	33.69	A#6-31	
8	36	C7	double octave
10	39.86	E7 -14	
10.76	41.13	F7 +13	
11	39.51	F#7-49	
13	44.40	A7 +40	
14	45.69	A#7-31	
14.38	46.15	A37 + 15	
16	48	C8	triple octave
17	49.05	C#8 +5	
19	51	D#8	
19.14	51.10	D#8 + 10	
20	51.86	E8 -14	
22	53.51	F#8-49	
22.76	54.10	F#8 + 10	
27.52	57.39	A8 +39	
31.14	59.53	C9 -4	
35.90	62	D9	
39.52	63.65	E9 -35	

A more convincing simulation can be obtained by extension with an additional FM pair carrier and modulator with a c:m ratio of 2:6. This results in a frequency spectrum that is one octave higher than the spectrum formed. by c:m = 1:3. In part this results in the same partial frequencies, which overlap and are emphasized (marked green).

The sideband frequencies that are formed by an FM pair with c:m ratio 2:6

Partial ratio	Offset: semitones.cents	Note name.cents	Name bell partial
2	12	C5	prime
4	24	C6	double octave
8	36	C7	triple octave
10	39.96	E7 -14	
14	45.69	A#7-31	
16	48	C8	quadruple octave
20	51.86	E8 -14	
22	53.51	F#8-49	
26	56.40	A8 +40	
28	57.69	A#8-31	
32	48	C9	
34	61.05	C#9 +5	
38	63	D#9	
40	63.86	E9 -14	
44	65.51	F#9-49	

The three spectra combined: the 6-oscillator FM bell synthesis

Partial ratio	Offset: semitones.cents	Note name.cents	Name bell partial
1	0	C4	fundamental
2	12	C5	prime
2.38	15	D#5	minor third
4	24	C6	octave
5	27.86	E6 -14	major tenth
6	31.02	G6 +2	fifteenth
7	33.69	A#6 -31	
8	36	C7	double octave
10	39.86	E7 -14	
10.76	41.13	F7 +13	double fourteenth
11	39.51	F#7-49	
13	44.40	A7 +40	
14	45.69	A#7-31	
14.38	46.15	A#7 + 15	
16	48	C8	triple octave
17	49.05	C#8 +5	
19	51	D#9	
19.14	51.10	D#9 + 10	
20	51.86	E9- 14	
22	53.51	F#9 -49	
22.76	54.10	F#8 + 10	
26	56.40	A8 +40	
27.52	57.39	a5 +39	
28	57.69	A#5 -31	
31.14	59.53	C9 -4	
32	60	C9	
34	61.05	C#9 +5	
35.90	62	D9	
38	63	D#9	
39.52	63.65	E9 -35	
40	63.86	E9 -14	
44	65.51	f#6-49	

For comparison, a sound analysis of a physical carillon bell

Partial number	Partial name	Pitch.offset cents	Volume	Decay time %
1	fundamental	C4 +31	mf	100
2	prime	C5 +31	f	55
3	minor third	D#5 + 40	ff	75
4	fifth	G5 +23	mp	20
5	octave	C6 +31	fff	30
6	major tenth	E6 +58	p	
7	1st fourteenth	F6 -34	p	
8	2nd fourteenth	F6 -14	p	
9	fifteenth	g3 +9	ff	20
10		A6 +2	pp	
11		B6 +32	pp	
12	double octave	C7 + 91 f		15
13		C#7 + 13		
14		C#7 + 22		
15		D7 +45		
16		D#7 + 29		
17		E& +60		
18	double fourteenth	F7 +56	mf	10
19		F#7 + 57		
20		F#7 + 64		
21		G7 +11		
22		G7 +31		
23		G#7 + 65		
24		A7 +18		
25		A7 +46	mp	7.5
26		A#7-7		
27		A#7 + 16		
28		A#7 + 36		

29		B7 -6		
30		B7 +53		
31		C8 -30		
32		C8 -11		
33		C8 +17		
34	triple octave	C8 +82	mp	5
35		C#8 + 12		
36		C#8 + 37		
37		C#8 + 58		
38		C#8 + 60		
39		D8 -4		
40		D8 +29		

After André Lehr from: 'Leerboek der Campanologie'
The analysis was based on a bell with the fundamental G#1+31 cent. For the sake of simplicity, and comparison with the other tables, I transposed the entire spectrum up to C4)

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