Deceptive Overtones

One spectrum: three different sounds; and why the size of the Tubular Bells is usually within two octaves.

The missing fundamental
The fundamental is missing, and yet you hear it! For example, if you play music and listen on your laptop or on the mini multimedia speakers of your desktop computer. You hear it all fine, while the format of the speakers already indicates that there is no or hardly any bass.

How did that happen? The fundamental may be removed by the filtering effect (high pass) of the mini speakers, the higher overtones are displayed.

In the case of sounds of wind and string instruments, this concerns harmonic overtones. Frequencies that are an integer multiple of the fundamental frequency. Because of this harmonic relationship, the starting points of these sine wave vibrations coincide at certain moments.

Let's take the example below of two frequencies in the ratio of 2 to 3, so a fifth interval. It becomes more insightful when we transpose this frequency ratio down to the time domain. We have then transformed these two simultaneous sounding frequencies, a fifth interval, to a rhythm: 2 to 3 to be precise as schematically shown below:

(3)	X	X		X	X	(right)
(2)	X		X		X	(left)
(P)	**X**				**X**	(r & l)

The time axis represents from left to right. The top line shows that there are three events (x) in the same time as two of them on the middle line. The bottom line indicates where both come together.

Where these two coincide, a new regularity arises, a new periodicity. You can find these on the highest common denominator (GGD) of the two periodicities (Jan Schouten, 1939). If you play this rhythm with both hands, this new periodicity arises where both hands together make a stroke movement at the same time.

In the pitch domain this is called virtual pitch, missing fundamental or residual pitch. Back to the sounding example of the fifth interval: the virtual pitch experience will now be one octave *lower* than the lowest tone of the fifth sound.

Missing Fundamental

Load `Mis-Fundament.pch2`. In this patch you can hear 8 different examples of virtual pitch, missing fundamental. All sample sounds are additive made up of four constituent frequencies that are generated with 4 OscCs. These are mixed in a mixer module, Mix4-1B.

As an extra, there is a fifth oscillator Osc-C connected to the mixer that you can use as a reference for comparison. The combined oscillator signals go to a FiltNord module in Low Pass mode. This filter is controlled by a decaying envelope, EnvADR. This simulates an decaying sound: After the attack, the filter frequency is modulated down by the envelope output, with the result that first the higher overtones disappear and the lower ones later.

Variations 1 to 5 show successively the missing fundamental built up from respectively higher successive partial frequency ratios.

Variations 6 and 7 provide examples of the missing fundamental made up of only odd harmonics. The final example, variation 8, shows that the missing fundamental also arises even when the constituent frequencies are quasi-harmonic: the frequency ratios are an approximation of 2, 3, 4 and 5 (1.9946, 2.9966, 4.0254 and 5.0397).

Our perception is apparently looking for (approximate) harmonic frequency relationships. Sounds composed of (quasi) harmonics result in fusion, an indivisible identity with unambiguous pitch, as is the case with sounds of wind and string instruments.

Plucked tubes and bars: one spectrum, three different sounds, three different pitches !?
The spectrum of an excited tube or bar, the ratios of the first seven resonance frequencies: 1, 2.76, 5.40, 8.93. 13.34, 18.84, 31.87.
These frequency ratios appear superficially random. Load the `Tube-Bar-Spectrum.pch2` patch and listen to this mix of frequencies.

Play this patch chromatically up and down over at least four octaves and be amazed at what happens with pitch and timbre perception. You don’t hear a consistent pitch and a fixed timbre over the entire range.

This sound simulation, however, is a true-to-life reproduction of the natural frequencies of an excited rod or tube with free ends. The only difference with reality is that in this synthesis all frequencies show the same amplitude. In the real tube chime or rod game, this is related to the length of the rod or tube and the mallet used: soft-hard, large-small. A large soft mallet will mainly activate the low resonant frequencies, a small hard mallet mainly the higher natural frequencies. With a very long tube the absolute frequency spectrum will be (much) lower than with a very short tube or rod.

In the patch I intentionally gave all partials the same amplitude. This makes it all the more clear that the loudness of the sub-tones plays an important role in the pitch and timbre perception of the sound. Our hearing is most sensitive to frequencies between 3000 and 4000 Hz. That means that depending on whether we play a high or low tone, certain partial tones fall into this most sensitive area.

For a very high tone played on our virtual mallet instrument, it is the lowest frequency that sounds the loudest. We experience this as 'fundamental' and determine the pitch.

However, if we play a very low tone on the virtual keyboard, it is precisely the higher overtones that fall in the most sensitive frequency range.
The table below shows that more or less three different perception models can play a role, depending on which sub-frequencies fall into the most sensitive part of our hearing.

Eigenfrequenties of a tube with free ends

1	2.76	5.40	8.93	13.34	18.64	31.87	**1)**

Glockenspiel model

1	2.76	5.40	8.93	13.34	18.64	31.87

Eigenfrequenties of a tube with free ends

1	2.76	5.40	8.93	13.34	18.64	31.87

2) Ambigu model

0.36 ***1***	**2**	3.23	4.83	6.75	11.55	

Eigenfrequenties of a tube with free ends

1	2.76		5.40	8.93	13.34	18.64	31.87

3) Tubular Bells model

0.21	0.6	***V1***	1.2	**2**	**3**	**4**	**7**

The Glockenspiel model
With short rods or tubes as we find them in the bar games from the Orff school musical instruments, we interpret the sound via the Glockenspiel model. The lowest partial acts as a 'fundamental'. The higher overtones only play a supporting role with a coloring effect.

The Tubular Bell model
Now the other extreme. For example, the tubular bells from the orchestral instruments. The tubes are relatively long with this instrument. The partial tones that fall in the most sensitive frequency

range are now the partials 4, 5, 6 and 7. If we take a closer look at the frequency ratios of these partial tones, we see that they show approximately proportions 2:3:4:7. Approximately integer multiples of a virtual frequency with a ratio of approximately 4.5. That is the pitch at which we experience the sound (v1).

Minor third

The other partial tones, number three with ratio number 5.4, is about a minor third higher than the virtual root (4.5) (4.5 is up to 5.4 is about 5 to 6, or simplified 1:1.2.) This minor third is an important characteristic of the (carillon) bell sound.

The second partial of the spectrum with a ratio of 2.76 appears to be about one octave lower than the third partial 5.40. We experience this partial as the low humming tone which also occurs in the real bell sound. Be it that in the real bell this humming tone is one octave lower than the 'strike tone'.

The lowest partial frequency of the tubular bell is so low and weak that it is negligible for the perception. A long time ago, experimenting in the percussion room of the Rotterdam Conservatoire, it turned out that you could obediently detect this lowest partial frequency by hearing the ear very close to the end of the tube.

The Ambigu model

For pitches played in the middle of our virtual keyboard, both the timbre and the pitch are rather dubious. For convenience, I have called this the 'Ambigu model'.
In the patch `1Spectrum-3Timbres.pch2` the two observation models Ambigu and Tubular Bells were transposed for comparison so that the pitch is equal to that of the Glockenspiel model. Why the size of the tubular bells mostly don't exceed a maximum one and a half octave will be clear now. If you increase the size, the instrument will inevitably transform itself in timbre and transpose itself in terms of pitch.

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Literature

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