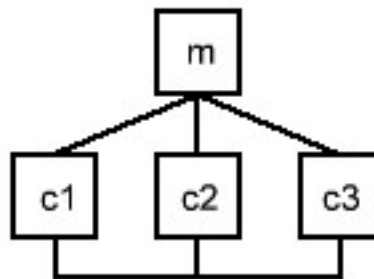


## FM Synthesis III, complex FM

The most simple form of complex FM is an operator configuration with multiple carriers modulated by only one modulator. The resulting spectrum we will find simply by applying the known formula for every carrier and modulator.



$$| c_1 \pm nm |, | c_2 \pm nm | \text{ and } | c_3 \pm nm |$$

This configuration is especially useful for the generation of sounds with clearly distinct formants.

### parallel modulation

Parallel modulation arises when a carrier is modulated by multiple modulators. As may be expected, the spectrum formed, in this case even more complex than in the basic configuration, simple FM.

We now have to apply the familiar formula  $| c \pm nm |$  for both modulator 1 and modulator 2. This then results in:

$$| c \pm nm_1 | \text{ and } | c \pm nm_2 |$$

In addition, however, there are also side bands formed by the modulation of the sum and difference frequencies of the various modulators.

In the example below, the following frequencies occur.

given:  $m_1 = 2$ ,  $m_2 = 11$ ,  $c = 1$

carrier  $c = 1$

$$| c + nm_1 | \quad 3, 5, 7, 9, 11, 13, 15, \dots$$

$$\begin{array}{lcl}
 | c - nm_1 | & 1, -3, 5, -7, 9, -11, 13, \dots \\
 \begin{array}{l} | c + nm_2 | \\ | c - nm_2 | \end{array} & \begin{array}{l} 12, 23, 34, 45, 56, 67, 78, \dots \\ 10, -21, 32, -43, 54, -65, 76, \dots \end{array} \\
 \begin{array}{l} | c + n(m_1 + m_2) | \\ | c - n(m_1 - m_2) | \end{array} & \begin{array}{l} 14, 27, 40, 53, 66, 70, 92, \dots \\ 8, -17, 26, -35, 53, -62, 71, \dots \end{array}
 \end{array}$$

For sidebands generated by  $m_1$  and  $m_2$  we can find the amplitude according to the Bessel functions. However, for sidebands which are formed by  $n(m_1 \pm m_2)$  we can find the amplitude by multiplying the values of the relevant Bessel functions.

Parallel modulation with multiple sinusoidal modulators can be seen as a complex modulation waveform. For each frequency of this modulation signal, sum and difference frequencies with the carrier are generated. In addition, sidebands also occur for the sum and difference frequencies in the modulation signal with the carrier frequency.

### series modulation, modulator in series or cascade

Likewise, we obtain a complex spectrum by means of modulation with more than one modulator in cascade. For example, in the following algorithm.



Here we see that operator  $m_2$  is modulating operator  $m_1$ . The output spectrum of  $m_1$  contains the frequencies governed by the known formula  $| m_1 \pm nm_2 |$ .

These two modulators can be regarded as a pair of simple FM,  $m_1$  now

serves as carrier of  $m_2$ . The complex output of  $m_1$  now is the complex modulation waveform for the carrier.

Also according to this algorithm, the so-called combination side bands formed by the individual frequencies that make up the output signal of the modulator 2 exists. And, moreover, by the sum and difference frequencies of the partial frequencies in the signal of modulator 2. This series modulation is very similar to the previous parallel modulation. Because the modulators are arranged in series, the first modulator acts as a scaler of the second modulator.

For both parallel and cascade modulation the spectrum density cumulatively increases with the the number of modulators. We can ask now whether it makes sense to calculate very precisely every frequency in the resulting spectrum.

An experience rule of thumb: the greater the spectrum density the less importance of the individual frequencies. Much more important for the observation of the spectrum are perceptual characteristics as harmonic, non-harmonic or partially harmonic, even or odd overtones.

## FM, a summary of formulas

The carrier frequency is always the fundamental if:

$c : m = 1 : 1$ , and if  $m$  equals or is greater than  $2c$

where

$m$  = modulation frequency

$c$  = carrier frequency

The ratio of carrier and modulator frequency determines the spectrum as follows:

$| c \pm nm |$  for  $n = 0, 1, 2, 3 \dots$

The spectrum with the maximum harmonicity is formed by:

$c : m = 1 : 1$

All  $c : m$  relationships in the form:

$N : n = N : 1$  for  $N = 1, 2, 3, \dots$

produce the same harmonic spectrum, where the carrier is the  $N$ th harmonic in the spectrum.

The modulation index,  $I$ , we find from the following formula:

$$I = \Delta c : m$$

where  $\Delta c$  = maximum deviation in Hz of the carrier frequency

As a rule of thumb for the number of relevant sidebands for the perception:

$$n = I + 2$$

The larger the modulation index, the more dense the resulting spectrum.

Certain  $c : m$  ratios can generate a spectrum of the same category. This spectrum is then built up from a different carrier frequency. According to the following formula, we find, given a certain  $c : m$  ratio, all other  $c : m$  ratios which produce a spectrum that belongs to the same category:

$$| c \pm nm | : m$$