FM Synthesis II, simple FM, some experiments with the DX7 and concise theory

a quick and easy 'summation trick'

experiment 1

We choose the simple FM algorithm ('Voice Initialize') and set the carrier frequency to `ratio 2` and the modulator frequency to `ratio 7`.

We set the modulator output level to 75.

If we change the modulator output level, we hear how this is related with the amount of harmonics in the sound.

rule 1, the amount of side bands/overtones/harmonics

The modulator output level, modulation depth or the maximum deviation of the carrier frequency determines how many harmonics are formed.

experiment 2

Now we're going to change the modulator frequency number from 7 to 6. Notice how dramatically it affects the tonal character.

rule 2, *which* sidebands/overtones/harmonics

The possible overtone spectrum is determined by the ratio between carrier and modulatorfequentie.

the actual side bands due to a given `c : m ratio`

If we just want to know which sidebands can be formed, with neglection of their strength.

experiment 3

We again choose the simple FM algorithm from experiment 1 with `c : m ratio = 2 : 7` and modulator output level 75 (DX7).

At this output level of 75, four side band pairs are generated (see appendix). The spectrum now consists of:

1) the carrier frequency itself, the pitch of the key we press,

2) and the upper and lower side bands:

With the following 'summation trick' we obtain the upper side bands. Take the carrier ratio number and add the modulation ratio number. Add to this result again the the modulator ratio number number. Repeat and repeat.
This yields the following upper side bands:

`(2 +7 =) 9, (9 +7) = 16, (16 +7) = 23, (23 +7) = 30`

Likewise, we find the lower side bands. First we subtract the modulator ratio number from the the carrier ratio number. As a result we obtain now `-5`. Now we neglect the minus sign. Then we repeat the 'summation trick' as in the calculation of the upper side bands. This results in the following lower sidebands:

`(2-7) = 5, (5 +7) = 12, (12 +7) = 19, (19 +7) = 26`

Then all upper and lower side bands in order:

`2, 5, 9, 16, 19, 23, 26, 30`

The lowest frequency number, '2 ', is the reference for our perception, or in other words this forms the fundamental frequency. So we interprete this lowest frequency as a '1 '. Therefore every number in the resulting series must be divided by 2. Then we get:

1, *2.5*, *4.5*, 8, *9.5*, *11.5*, 13, 15

These numbers directly represent the frequency ratios of the formed overtone series. The result is a sound consisting of a mixture of harmonic overtones, the whole numbers, and inharmonic overtones, the numbers with decimals.

experiment 4

Now we are going change the carrier frequency sequentially to ratio `5, 9, 16, 23, 26` and `30`. We hear then successively related spectral frequencies, belonging to the same category.

rule 3, different `c : m ratios` can generate overtones belonging to the same category.

From a given `c: m ratio` we may replace the carrier ratio number by any number from the side band series.
In our example, `c : m = 2 : 7`, we may therefore replace the carrier ratio number `2`, by any other number from the side band series: `5, 9,`

16, 19, 23, 26, 30.
By replacing the carrier ratio number with any ratio number from the side band series we obtain catgorical related spectra to the spectrum of the basic `c : m ratio`.

simple FM, more accurately viewed: in formulas and some calculation examples

The `c : m ratio` determines which overtones, harmonics can be obtained, according to the next formula.

`| c ± nm |`

where:
c: carrier frequency
m: modulator frequency
n: number of the row of the integers `0, 1, 2, ...`

According to this formula sum and difference frequencies are generated. These sum and difference frequencies are called side bands in FM speak.

How many of these sidebands arise is determined by `I`, `the modulation index`, according to the following formula.

`I = mfdC : m`

`mfdC`: the **m**aximum **f**requency **d**eviation of the carrier frequency in Hz, under the influence of `m`, the amount of frequency of modulation.

According to the following rule of thumb, we find the relevant number of side band pairs, n.

`n = I + 2`

It all seems a bit more difficult than it actually is. Let's take a concrete example, and look at what these formulas mean in practice.

Suppose we have the following assumption.

```
modulator frequency, 100 Hz
number of sidebands, 4
carrier frequency, 1000 Hz
```

There now originate in the output signal, apart from the carrier frequency also sum and difference frequenties of carrier frequency and modulator frequency. The sum frequencties are the upper side bands and the difference frequencies are representing the lower side bands.

The number n indicates the number of side bands wich are relevant for the perception.

In case of four upper side bands it means that we must do the calculation four times as follows.

```
c + 1m = 1000 + 100 = 1100
c + 2m = 1000 + 200 = 1200
c + 3m = 1000 + 300 = 1300
c + 4m = 1000 + 400 = 1400
```

Also for the lower side bands we must make four similar calculations. However keep in mind that each odd lower side band must be multiplied by –1.

```
- (c - 1m) = - (1000 - 100) = -900
   c - 2m  =     1000 – 200  =  800
- (c - 3m) = - (1000 - 300) = -700
   c - 4m  =     1000 - 400  =  600
```

All neatly put in order including the carrier frequency we obtain the following list:

```
1400 Hz
1300 Hz
1200 Hz
1100 Hz
1000 Hz
-900 Hz
 800 Hz
-700 Hz
 600 Hz
```

With only two sinusoidal frequencies, we now generated a sound consisting of no less than nine resulting frequencies. By raising the output level of the modulator still more sidebands are formed.

Another example with carrier and modulator frequencies exchanged:

```
c = 100 Hz
m = 1000 Hz
n = 4
```

By applying the known formula | `c ± nm` | we find the following results.

```
c, the carrier frequency, 1000 Hz
```

the lower side band frequencies:

```
- (c - 1m) = - (100 - 1000) =   900
   c - 2m  =     100 - 2000  = -1900
- (c - 3m) = - (100 - 3000) =  2900
   c - 4m  =     100 - 4000  = -3900
```

the upper side band frequencies:

```
c + 1m = 100 + 1000 = 1100
c + 2m = 100 + 2000 = 2100
c + 3m = 100 + 3000 = 3100
c + 4m = 100 + 4000 = 4100
```

upper and lower side band frequencies in order:

```
900, 1100, -1900, 2100, 2900, 3100, -3900, 4100
```

Notice: these successive frequencies belong alternately to the lower and upper sidebands.

In the lower side band series we find negative frequencies. This simply means that these frequencies exhibit a phase shift of 180 degrees. In most cases we can neglect this phase shift.

intervals, frequency ratios

Because we do not recognise absolute frequencies, but intervals or frequency ratios, it is much easier to fill in the frequency ratios.
In the result of the formula we then find immediately the harmonic numbers in the spectrum.

The following example taken from the DX7 Operating Manual
`c : m = 1 : 3` results the following spectrum.

the carrier, root, first harmonic, 1

the lower side band harmonics:

```
- (c - 1 m) = - (1 - 3) =  2
   c - 2m   =    1 – 6  = -5
- (c - 3m)  = - (1 - 9) =  8
   c - 4m   =    1 -12 = -11
```

the upper side band harmonics:

```
c + 1m = 1 +  3 =  4
c + 2m = 1 +  6 =  7
c + 3m = 1 +  9 = 10
c + 4m = 1 + 12 = 13
```

How far this spectrum will be formed according to the known formula, of course, depends in turn on the modulation index. In practice, in a Yamaha FM synthesizer, this is determined by the modulator output level.

An important conclusion from our experiments. As the modulation depth increases, the carrier frequency, globally, becomes weaker while more side bands are growing. The total energy in the carrier signal, due to increasing modulation depth, remains constant however. This means that energy is 'stolen' from the carrier frequency and is distributed in the occuring sideband frequencies.
When we change the modultion depth dynamically by means of an envelope generator we obtain a gradient strength of the mutual partial frequencies.

This resembles the behaviour of filters in subtractive synthesis. Likewise we could relate the `c : m ratio` to the waveform in subtractive synthesis.

However, with the determination of the `c : m ratio`, we have a much more powerful tool for sound shaping available than in the subtractive variant, oscillator and filter. With a proper choice of the `c : m ratio` we may generate sounds with inharmonic frequencies. Such inharmonic spectra are essential for synthesizing sounds of percussion instruments like bells, metallophones, xylophones and membranophones.

It is possible to generate spectra of the same category with different `c : m ratios`.

With the following formula can we reduce each given `c : m ratio` down to its basic form.

```
c = | c - m |
```

For example, the `c : m ratio 13 : 5` is then simplified as follows.

```
c = | c -  m |

c = | 13 - 5 | = 8
c = |  8 – 5 | = 3
c = |  3 - 5 | = 2
```

Thus the basic form is `c : m = 2 : 5`

Yet there are differences between audible spectrum generated by these related `c : m ratios`. With the same modulation depth and related `c : m ratios`, the corresponding frequencies in the spectrum show different amplitudes.This is very clearly audible in the following experiment.

experiment 5

OP2 modulator EG DX7 data:
`output level 75`

```
R1 = 15, R2 = 15, R3 = 15, R4 = 15
L1 = 99, L2 = 99, L3 = 99, L4 = 00
```

OP1 carrier EG DX7 data:
`Output level 99`

```
R1 = 99, R2 = 99, R3 = 99, R4 = 99
L1 = 99, L2 = 99, L3 = 99, L4 = 00
```

We will now perform this experiment with the following `c : m ratios` with constant data for both envelopes and output levels as indicated.

`2:5, 3:5, 7:5, 8:5, 12:5 and 13:5`

In these various `c : m ratios` we hear a quite different time variant behaviour of timbre.

Although in all cases the same spectrum category is formed, the way in which this spectrum evolves in time by means of the modulator envelope is clearly different in perception.

At the time the modulation depth is still zero, or very small, only or mainly, the carrierfrquentie is present in the output signal. By increasing of the modulation depth, under the influence of the modulator envelope, the carrier will, however, always be softer in favour of the appearing sidebands. In this example, therefore, always the same category of spectrum is formed from a different carrier frequency.

By means of the following formula, starting from a given c : m ratio, all other c : m ratios can be calculated that the same spectrum.

$| c \pm nm | : m$

Let us consider the `c : m ratio`

$c : m = 1 : 2$

The above formula used yields the following result.

$| c \pm nm | : m = c : m$

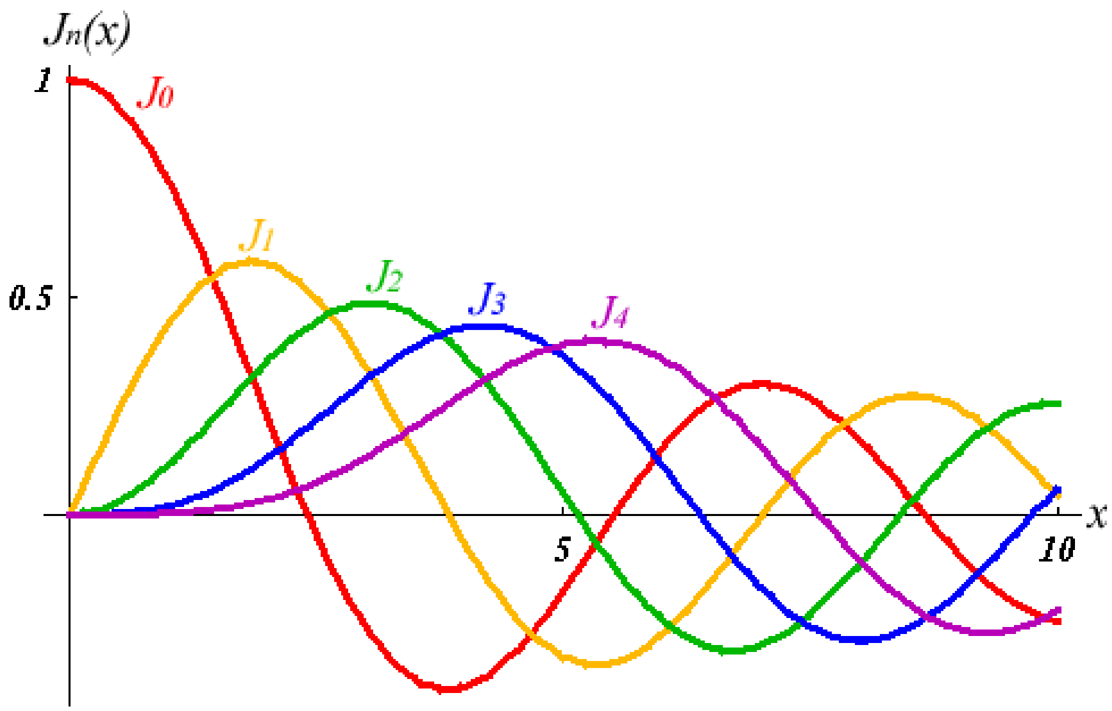
for $n = 1$

$| 1 + 2 | : 2 = 3 : 2$
$| 1 - 2 | : 2 = 1 : 2$

for $n = 2$

```
| 1 + 4 | : 2 = 5 : 2
| 1 – 4 | : 2 = 3 : 2


for n = 3

| 1 + 6 | : 2 = 7 : 2
| 1 – 6 | : 2 = 5 : 2

for n = 4

| 1 + 8 | : 2 = 9 : 2
| 1 – 8 | : 2 = 7 : 2

for n = 5

| 1 + 10 | : 2 = 11 : 2
| 1 – 10 | : 2 =  9 : 2

for n = 6

| 1 + 12 | : 2 = 13 : 2
| 1 – 12 | : 2 = 11 : 2

etc.
```

With increasing of the modulation depth, by raising the modulator output level, the carrier frequency became globally weaker and weaker. In the previous sentence *globally* is used, and not for nothing. If we have listened attentively we have also found that within this overall course of weakening of the carrier and the overall strengthening of the sidebands still finer modulations were obvious. Fluctuations as shown in the figure below. These curves are known in mathematics as Bessel functions.



In this figure (from Wikipedia), we see the curves that show how the amplitudes of the carrier and the sidebands fluctuate with increasing modulation index.

Bessel function J_0 shows the curve for the carrier. J_1 and J-consecutive numbers indicate how successive side band pair amplitudes increase and decrease.

For example, if we look at the the red function J_0, the curve for the amplitude fluctuation of the carrier, we notice that at the increase of the modulation index the amplitude becomes smaller and smaller and even decreases to zero.

Then the amplitude raises again, but this time with reverse phase. The red curve is now located below the x-axis. Thereafter the amplitude decreases to zero and then grows again with positive sign.

In a slowly increasing modulator output level, under the influence of a slow attack rate and high modulator output level, these 'volume garlands' are an ear catching characteristic of FM synthesis. If we want to avoid this characteristic, it is important the to keep the modulation index small.

feedback

The feedback algorithm may be considered as a mini algorithm that corresponds to the basic simple FM algorithm with `c : m = 1 : 1`. Provided, however, that in the feedback loop no separate envelope data can be entered. Only the amount of feedback level may be entered from 0 to 7. The output of the modulator is fed back directly to its own modulation input. This is called self-modulation.

Feedback was not part of Chowning's original FM model, but an addition of Yamaha. It is a major extension to the original simple FM model.

The amplitude levels between the carrier and sidebands are changing rather whimsical according to the the modulation index.

By applying feedback a much more evenly ampitude course between the carrier and the sidebands occurs. With a certain adjustment of the feedback level, we can generate a signal that in terms of waveform, and spectrum concerned, is almost identical to a sawtooth waveform.